Edexcel Maths FP1

Topic Questions from Papers

Numerical Solutions

Leave blank

5. $$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.2].

(2)

(b) Find $f'(x)$.

(3)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

Leave blank

4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

(a) show that $2.2 < \alpha < 2.3$

(2)

(b) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

Leave blank

2.

$$f(x) = 3x^2 - \frac{11}{x^2}$$

(a) Write down, to 3 decimal places, the value of f(1.3) and the value of f(1.4).

(1)

The equation f(x) = 0 has a root α between 1.3 and 1.4

(b) Starting with the interval [1.3, 1.4], use interval bisection to find an interval of width 0.025 which contains α.

(3)

(c) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

Leave blank

3.

$$f(x) = x^3 - \frac{7}{x} + 2, \quad x > 0$$

(a) Show that $f(x) = 0$ has a root α between 1.4 and 1.5

(2)

(b) Starting with the interval $[1.4, 1.5]$, use interval bisection twice to find an interval of width 0.025 that contains α.

(3)

(c) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

Leave blank

3. $$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$.

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α. Give your answer to 3 decimal places.
(4)

(b) Differentiate $f(x)$ to find $f'(x)$.
(2)

(c) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.
(4)

Leave blank

1. $$f(x) = 3^x + 3x - 7$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(b) Starting with the interval $[1, 2]$, use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

Leave blank

4. $$\mathrm{f}(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$$

(a) Use differentiation to find $\mathrm{f}'(x)$.

(2)

The root α of the equation $\mathrm{f}(x) = 0$ lies in the interval $[0.7, 0.9]$.

(b) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

Leave blank

2. (a) Show that $f(x) = x^4 + x - 1$ has a real root α in the interval $[0.5, 1.0]$.

(2)

(b) Starting with the interval $[0.5, 1.0]$, use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

(c) Taking 0.75 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places.

(5)

Leave blank

3. $$\mathrm{f}(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$$

A root α of the equation $\mathrm{f}(x) = 0$ lies in the interval $[3, 5]$.

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

Leave blank

6. $$f(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$.

(2)

(b) Use linear interpolation once on the interval $[1, 2]$ to find an approximation to α. Give your answer to 2 decimal places.

(3)

Leave blank

3.

$$f(x) = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}} - 5, \qquad x > 0$$

(a) Find $f'(x)$.

(2)

The equation $f(x) = 0$ has a root α in the interval [4.5, 5.5].

(b) Using $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

Leave blank

2. $$f(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(b) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

Leave blank

3. $$\mathrm{f}(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α between $x = 2$ and $x = 2.5$

(2)

(b) Starting with the interval $[2, 2.5]$ use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $\mathrm{f}(x) = 0$ has a root β in the interval $[-2, -1]$.

(c) Taking -1.5 as a first approximation to β, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$